

Introduction

Debate: whether instructional-based teaching or exploration-based active learning is better.

Obstacle: hard to control and characterize pedagogical methods and learning conditions in empirical studies (Prince 2004).

Our approach: compare computational models of teaching and active learning to formalize the methods and learning process.

Ideal learner: perfect memory, observation, and reasoning skills.

Concept spaces are hierarchical, can be partially ambiguous (e.g., “bird” and “sea animal” as concepts), and can be mismatched between the learner and teacher.

Questions: under what conditions is teaching better than active learning and vice versa?

Formal definitions

x : features (upper left, upper right, lower left, lower right)

y : binary labels (black or white)

H : concept space (has two levels of hierarchy)

h : higher level concept (always two of them, h_1 and h_2)

f : lower level pattern (1 to 6 within each h)

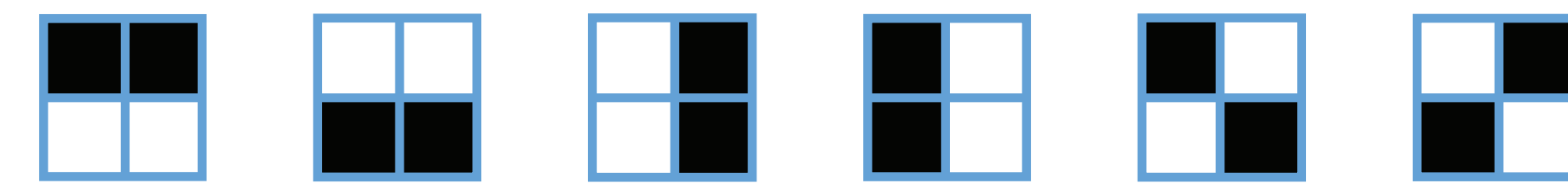
a : degree of ambiguity—the number of shared patterns between h_1 and h_2 .

m : degree of misalignment—the minimum number of pattern “moves” within a concept space to make two concept spaces equivalent.

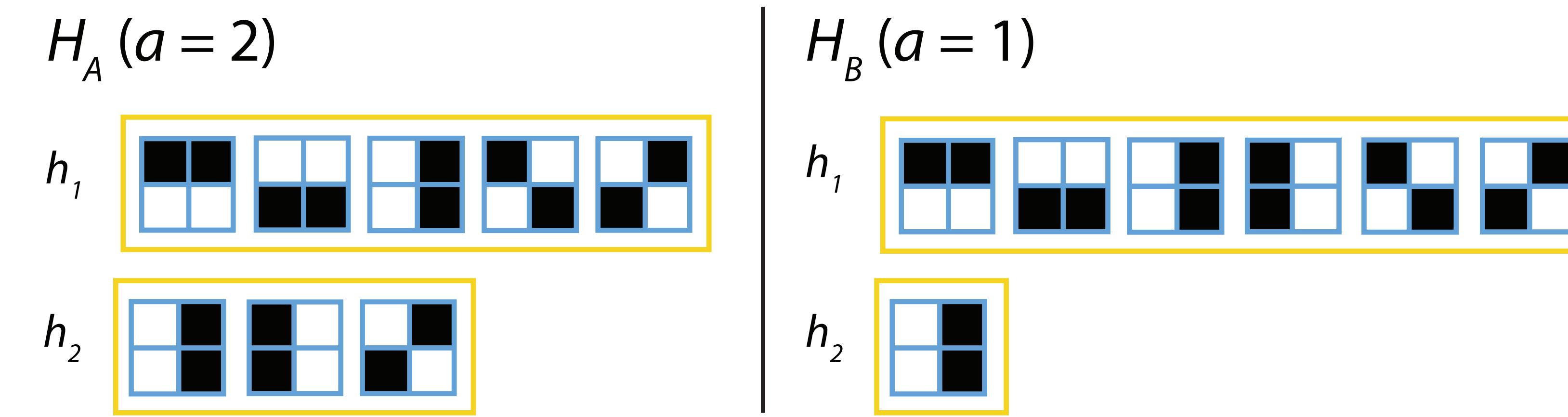
H_W, H_T, H_L : the true concept space, the teacher’s concept space, and the learner’s concept space, respectively.

Concept space

4 binary features forming 6 possible patterns:



Two example concept spaces ($m = 2$ between them):



Models

Active learning: choosing the feature that will maximize the expected reduction in uncertainty (MacKay 1992).

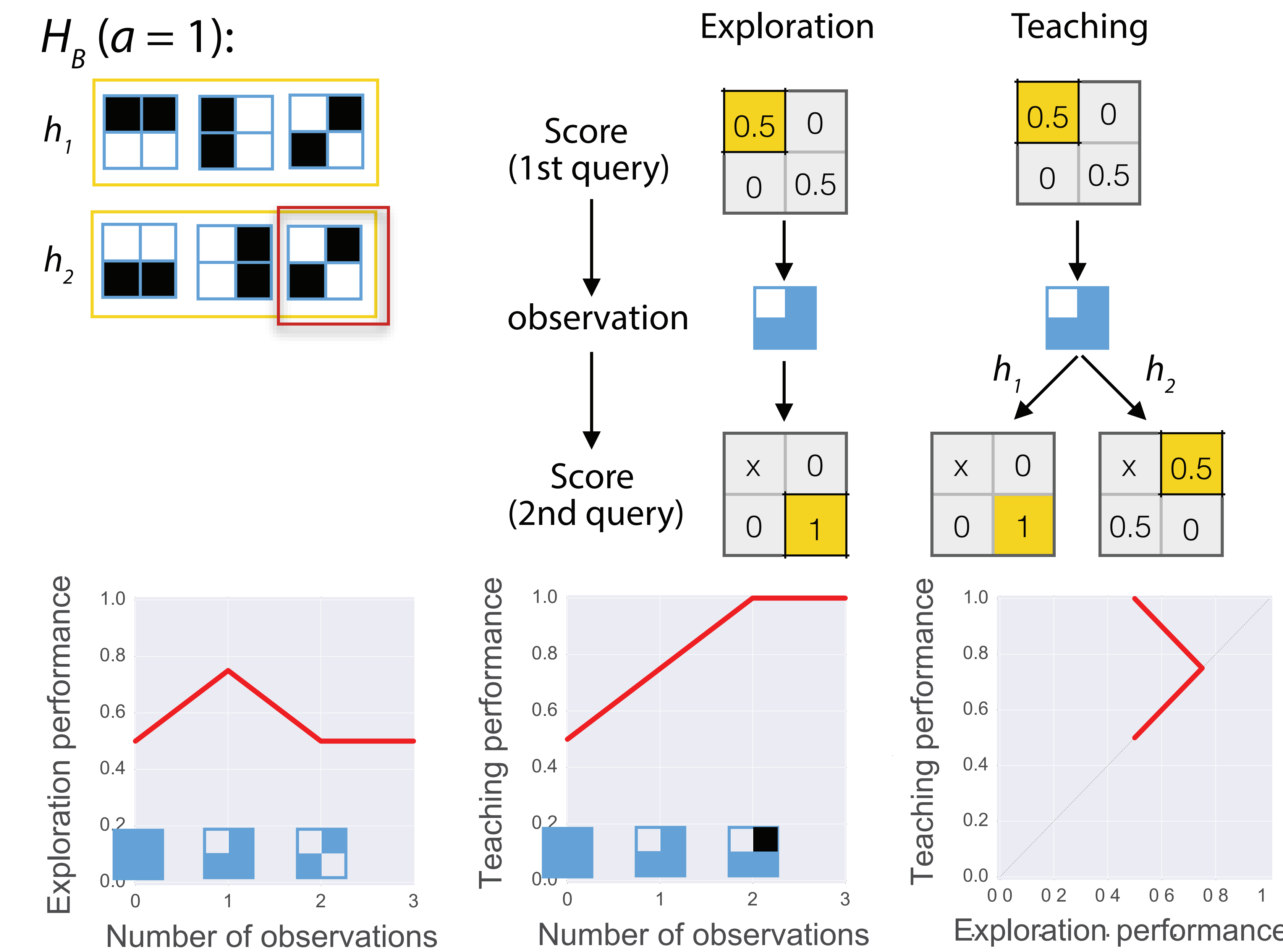
$$P(x|D) = \lim_{\alpha \rightarrow \infty} \frac{1}{Z} \left[\frac{\text{uncertainty of concept posterior before new observation}}{\text{uncertainty of concept posterior after new observation}} \frac{\text{expectation}}{\langle H[h|D] - H[h|x, y, D] \rangle_{P(y|x, D)}} \right]^\alpha$$

Teaching (cooperative inference): teacher knows the answer and reasons about the learner; learner reasons about how such teacher chooses the most helpful guidance (Shafto et al. 2014).

$$P_L(h|x, y, D) = \frac{1}{Z} \sum_j P(y|x, f_j) P_T(x|h, D) P_L(h, f_j|D)$$

$$P_T(x|h, D) = \lim_{\alpha \rightarrow \infty} \frac{1}{Z} \left[\frac{\text{learner's updated inference}}{\text{observation likelihood}} \frac{\text{learner's inference}}{\langle P_L(h|x, y, D) \rangle_{P_L(y|x, h, D)}} \frac{\text{teacher's selection posterior}}{P_T(x)} \frac{\text{teacher's selection prior}}{\langle P_L(h|x, y, D) \rangle_{P_L(y|x, h, D)}} \right]^\alpha$$

Example



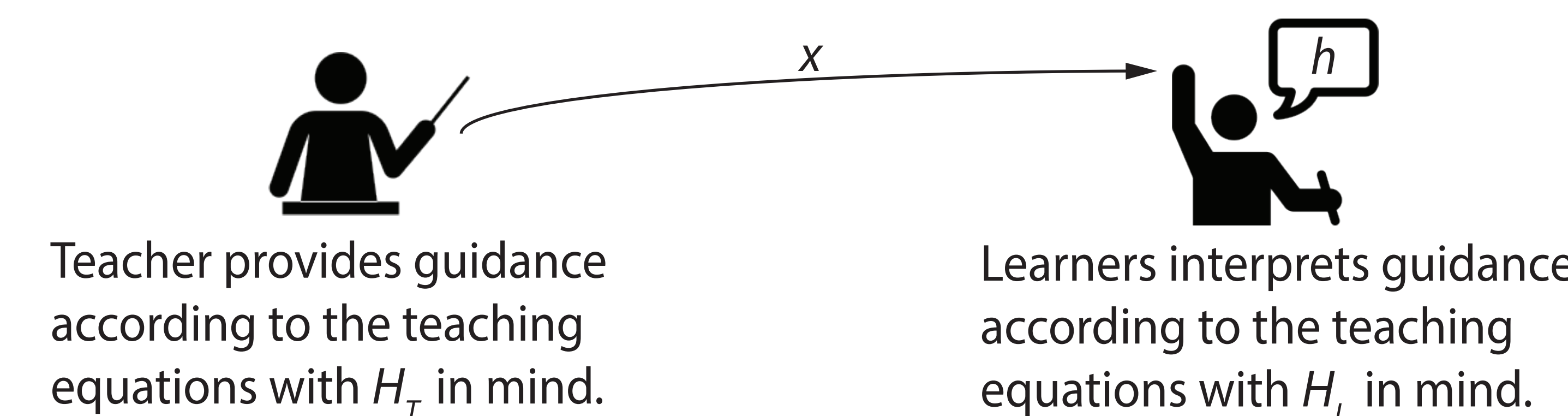
Conceptual misalignment

Assumptions of teaching:

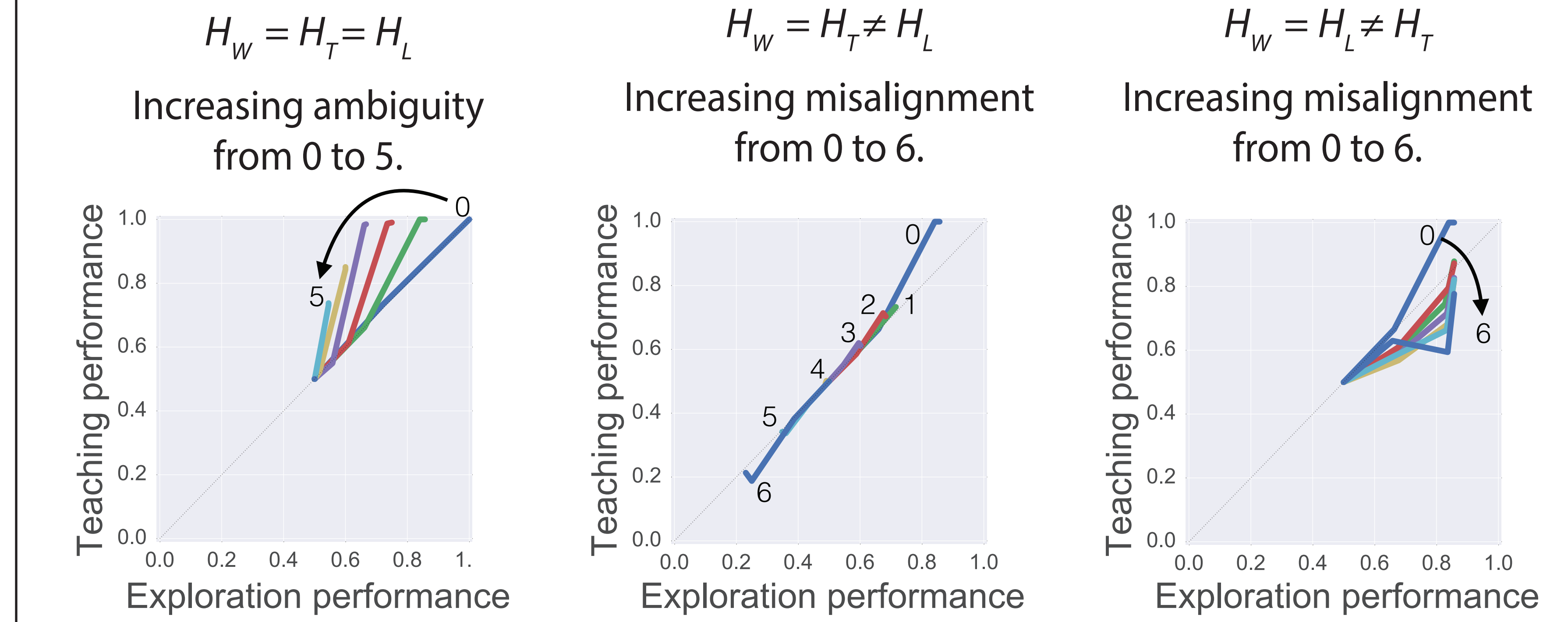
- (1) teacher knows the correct answer
- (2) $H_W = H_T = H_L$, and teacher and learner use exactly the same inference scheme.
- (3) teacher and the learner are cooperative.

Keep assumptions (1) and (3); break assumption (2) in two ways:

- misconception in learner ($H_W = H_T \neq H_L$)
- misconception in teacher ($H_W = H_L \neq H_T$)



Results



Conclusions

- 1) When both methods are at their best, teaching is better because it *reduces irrelevant search* and because of *cooperative inference*.
- 2) If one knows little about the structures and alignments of H_W , H_T , and H_L , teaching is preferred because it can potentially be much better and will unlikely be much worse than active learning.
- 3) If one knows that there is moderate amount of misalignment, active learning is the preferred method.

References

- MacKay, D. J. (1992). Information-based objective functions for active data selection. *Neural computation*, 4(4), 590–604.
- Prince, M. (2004). Does active learning work? A review of the research. *Journal of engineering education*, 93(3), 223–231.
- Shafto, P., Goodman, N. D., & Griffiths, T. L. (2014). A rational account of pedagogical reasoning: Teaching by, and learning from, examples. *Cognitive psychology*, 71, 55–89.

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